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Modelling

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Modelling of Corrosion Cracks

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MODELLING CORROSION CRACKS

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Abstract Modelling of corrosion cracking of reinforced concrete structures is complicated as a great number of uncertain factors are involved. To get a reliable modelling a physical and mechanical understanding of the process behind corrosion cracks is needed.

Corrosion cracks are due to the increased volume of corrosion products. After corrosion initiation the rust products from the corroded reinforcement will initially fill the porous zone near the reinforcement and then result in an expansion of the concrete near the reinforcement. Tensile stresses are then initiated in the concrete. With increasing corrosion the tensile stresses will at a certain time reach a critical value and cracks will be developed.

The increase of the crack width after formation of the initial crack is the subject of this paper. A recent model for crack propagation is reviewed and new results based on a Finite Element Analysis (FEM) are presented.

Keywords: Mathematical modelling, Finite element analysis, Corrosion cracks, Reinforced concrete.

Introduction

Important developments have taken place in the field of deterioration of reinforced concrete structures during the last 20 years; see e.g. [3]. Methods of modelling of deterioration of reinforced concrete structures can be divided into three broad classes:

Level 3: Scientific based level.

Level 2: Engineering level.

Level 1: Technical level.

Level 3 is an advanced scientific level based on an exact modelling of the deterioration profile (deterioration rate as a function of the time). Ad-

vanced information on the concrete microstructure is used to model e.g. the diffusion of chloride ions into the concrete. Further, advanced information on the environmental loading of the structure is used to model e.g. the time dependency of the chloride concentration on the surface of the structure. Therefore level 3 modelling of the deterioration is oriented at single structures. Level 3 models are based on a full probabilistic description of the various quantities which affect the deterioration of the concrete structure.

Level 2 is an advanced engineering level based on average material parameters such like the diffusion coefficient. Level 2 modelling is based on average loading parameters e.g. the chloride concentration on the surface and on simplifications regarding the ingress such as using Ficks law. The deterioration modelling is limited to a single or few deterioration mechanisms like chloride induced corrosion of the reinforcement. Level 2 models can be used for design of new structures, but also for assessing deterioration rates for groups of existing structures.

Level 1 is a simplified technical level based on direct observations of the deterioration mechanisms. It is based on a limited number of parameters e.g. obtained from level 2 modelling. Level 1 modelling is often used on groups of structures to obtain e.g. optimal maintenance strategies.

1. State of the Art

In this paper only chloride induced corrosion of the reinforcement is considered. If the rate of chloride penetration into concrete is modelled by Ficks law of diffusion, then it can be shown that the time T_{corr} to initiation of reinforcement corrosion is

$$T_{corr} = \frac{d^2}{4D} \left(\text{erf}^{-1} \left(\frac{C_{cr} - C_0}{C_i - C_0} \right) \right)^{-2} \quad (1)$$

where d is the concrete cover, D is the diffusion coefficient, C_{cr} is the critical chloride concentration at the site of the corrosion, C_0 is the equilibrium chloride concentration on the concrete surface, C_i is the initial chloride concentration in the concrete, erf is the error function. All the parameters mentioned above are modelled by stochastic variables or stochastic processes.

After corrosion initiation the rust products will initially fill the porous zone around the steel/concrete surface caused by the transition from paste to steel and entrapped/entrained air voids and then result in an expansion for the concrete near the reinforcement. As a result of this, tensile stresses are initiated in the concrete. With increasing corrosion the tensile stresses will reach a critical value and cracks will be devel-

oped. During this process the volume of the corrosion products at initial cracking of the concrete W_{crit} will occupy three volumes, namely the porous zone W_{porous} , the expansion of the concrete due to rust pressure W_{expan} , and the space of the corroded steel W_{steel} . With this modelling and some minor simplifications it can then be shown that the time from corrosion imitation to crack initiation is; see [2]

$$\Delta t_{crack} = \frac{1}{2 \times 0.383 \times 10^{-3} D_{bar} i_{corr}} \times \left(\frac{\rho_{steel}}{\rho_{steel} - 0.57 \rho_{rust}} (W_{porous} + W_{expan}) \right)^2 \quad (2)$$

where D_{bar} is the diameter of the reinforcement bar, i_{corr} is the annual mean corrosion rate, ρ_{steel} is the density of the steel, and ρ_{rust} is the density of the rust products. It is in the derivation of (2) assumed that the diameter $D_{bar}(t)$ of the reinforcement bar at the time t is modelled by

$$D_{bar}(t) = D_{bar}(T_{corr}) - c_{corr} i_{corr} (t - T_{corr}) \quad (3)$$

where c_{corr} is a corrosion coefficient.

After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the crack width w_{crack} is increased. Experiments show that the function between the reduction of the rebar diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} in a given time interval Δt measured on the surface of the concrete specimen can be approximated by a linear function

$$\Delta w_{crack} = \gamma \Delta D_{bar} \quad (4)$$

where the factor γ is of the order 1.5 to 5. It follows from (3) and (4) that

$$\begin{aligned} w_{crack}(t) &= w_{crack}(T_{crack}) + \gamma (D_{bar}(T_{crack}) - D_{bar}(t)) \\ &= w_{crack}(T_{crack}) + \gamma c_{corr} i_{corr} (t - T_{crack}) \end{aligned} \quad (5)$$

Let the critical crack width be $w_{critical}$ occurring at the time $T_{critical}$. By setting $w(T_{critical}) = w_{critical}$ the following expression is obtained for $T_{critical}$

$$T_{critical} = \frac{w_{critical} - w_{crack}(T_{crack})}{\gamma c_{corr} i_{corr}} + T_{crack} \quad (6)$$

$w_{crack}(T_{crack}) \approx 0$ is the initial crack width at the time T_{crack} .

2. Experimental Verification

Several researchers have investigated the evolution of corrosion cracks in reinforced concrete beams experimentally. After formation of the initial crack the rebar cross-section is further reduced due to the continued corrosion, and the width of the crack w_{crack} is increased. In the experiments an impressed current are normally used to artificially corrode the beams. The loss of bar sections is then monitored and the corresponding crack evolution is measured by the use of strain gauges attached to the surface of the beams. In most experiments the function between the reduction of the rebar diameter and the maximum crack width measured in the surface of the concrete specimen can be approximated by a linear function. Based on this linearization the increased crack width with time may be modelled.

The evolution of corrosion cracks in reinforced concrete beams has been experimentally investigated, see [1]. In the paper four simple test specimens have been investigated. The specimens are simplified small reinforced concrete beams with only a single rebar and 2 or 3 cm of cover. An impressed current artificially corrodes the beams. The loss of bar sections is monitored and the corresponding crack evolution is measured by the use of strain gauges attached to the surface of the beams. In all four experiments the function between the reduction of the rebar diameter and the maximum crack width measured in the surface of the concrete specimen can be approximated by a linear function, see figure 1.

Let Δw_{crack} be the increase in crack width in the time interval Δt and let the corresponding loss of rebar diameter be ΔD_{bar} . Then $\Delta w_{crack} = \gamma \Delta D_{bar}$, where γ is of the order 1.5 to 5 in the experiments reported in [1]. The factor depends on the cross-sectional data.

3. FEM Verification

The coefficient γ in equation (4) can for a given beam cross-section be estimated using FEM analysis. For illustration purposes, consider the rectangular beam cross-section shown in figure 2. The reinforcement consists of only one reinforcement bar. The diameter of the hole around the rebar at the time of crack initiation is $D_{hole} = 20$ mm and that the cover is $c = 10$ mm. The initial crack width is 0.1 mm.

In the FEM modelling the rectangular cross-section is assumed to have a hole at the site of the reinforcement and a crack from the hole to the boundary as shown in figure 2. The total triangular FEM net and the very fine local FEM net near the crack are shown in figure 2. The number of constant strain elements is 5580 and there are 3066 nodes.

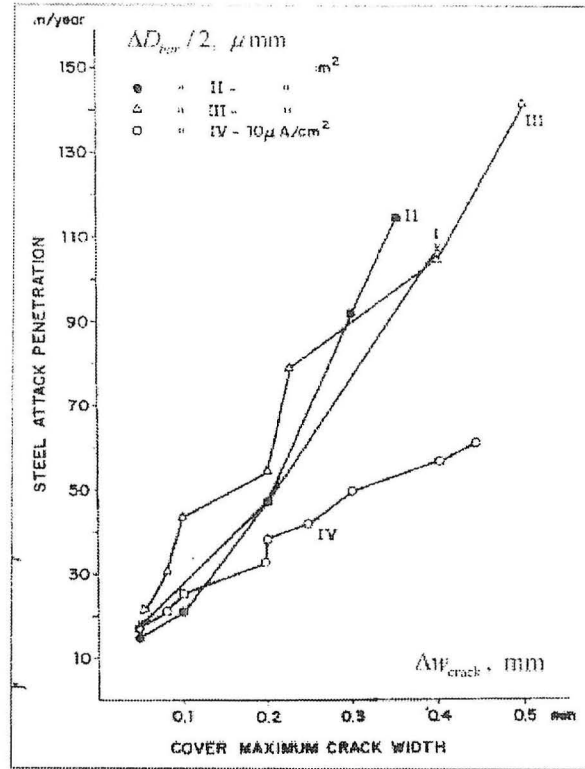


Figure 1. Loss in rebar diameter ΔD_{bar} versus the crack width Δw_{crack} . [1].

The FEM analysis is made using FEMLAB/MATLAB. The material is assumed to be linear elastic with the elasticity module $E = 25 \times 10^9$ Pa and the pressure from the increasing corrosion products is modelled as a uniform loading (pressure) $p = 1 \times 10^8$ N/m at the boundary of the hole.

The result of the analysis is shown in figure 3. To illustrate the procedure an (unrealistic) high pressure is used and the displacements are shown in figure 3 with an enlargement factor equal to 3. The increase in the crack width is $\Delta w_{crack} = 0.67$ mm and the average increase in the hole diameter is $\Delta D_{hole} = 0.31$ mm. Therefore

$$\eta = \frac{\Delta w_{crack}}{\Delta D_{hole}} = \frac{0.67}{0.31} = 2.2 \quad (7)$$

From $\Delta w_{crack} = \gamma \Delta D_{bar}$ and $\Delta w_{crack} = \eta \Delta D_{hole}$ it follows that

$$\Delta D_{bar} = \frac{\eta}{\gamma} \Delta D_{hole} \approx \Delta D_{hole} \text{ for small } \Delta w_{crack} \quad (8)$$

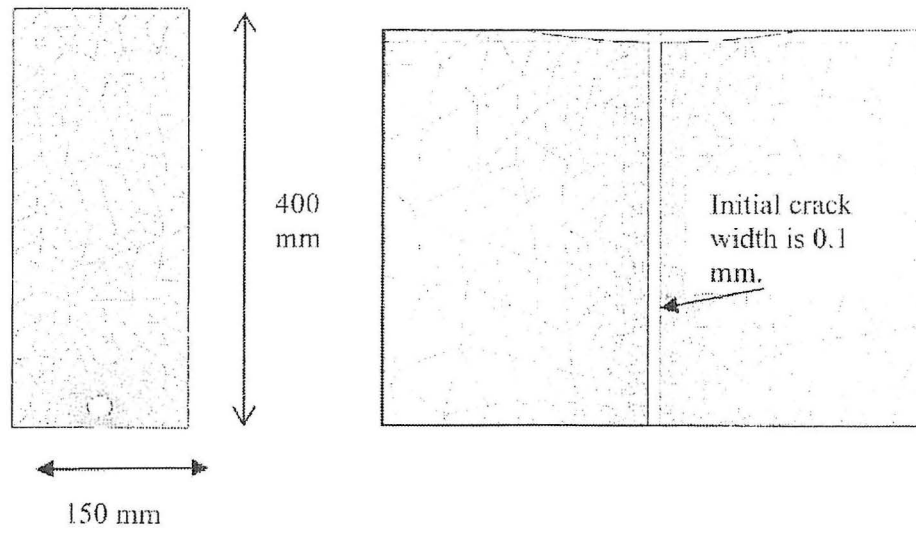


Figure 2. FEM net. The total net to the left and the local net near the crack to the right.

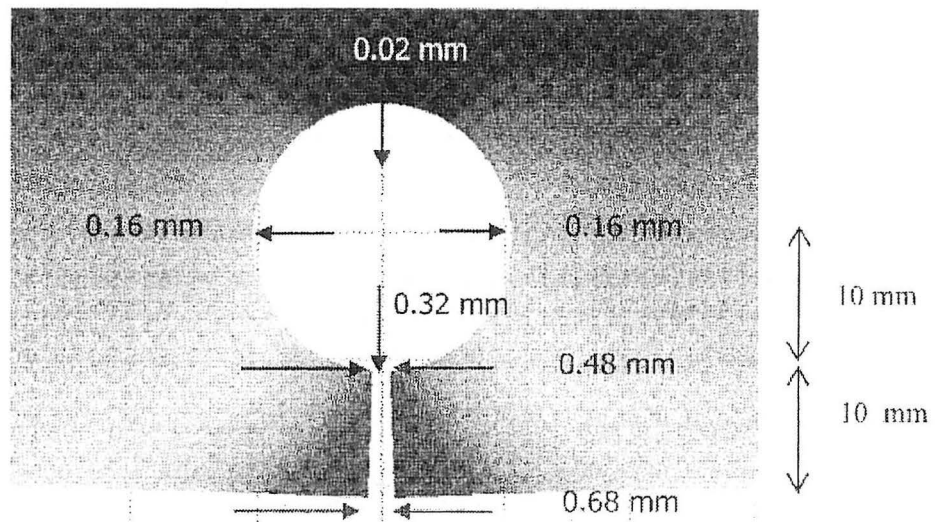


Figure 3. Displacements at four points of the circular hole and at the ends of the crack.

It follows from (8) that

$$\gamma \approx \eta \text{ for small } \Delta w_{crack} \quad (9)$$

so that the finite element estimation of η can be used to estimate the reduction of Δw_{bar} . In the next section is shown an alternative way to estimate Δw_{bar} .

4. Geometrical Approach

A simple approximate estimation of the coefficient γ in (4) for a given beam cross-section may be obtained by the procedure presented below. For illustration, assume that the diameter of the rebar at the time of crack initiation is $D = 16$ mm and that the cover is $c = 30$ mm, see figure 4.

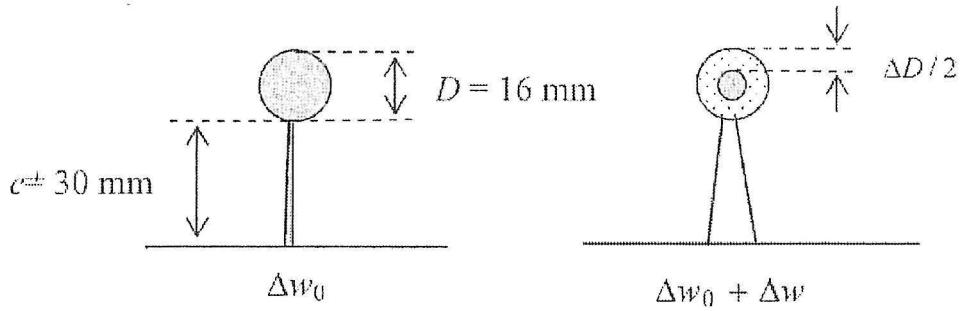


Figure 4. Evolution of cracks.

The crack at the time of the initial crack is shown to the left in figure 4. Let the initial crack width be Δw_0 . To the right the assumed crack configuration at the time when the diameter loss is ΔD is shown. The crack width is now $\Delta w_0 + \Delta w$. The procedure is then based on the assumption that the increase in the volume (area) of the crack is equal to the volume (area) of the corrosion products produced when the diameter is reduced to $D - \Delta D$. The relationship between Δw and D can then be obtained approximately by

$$\frac{1}{2} \left(\frac{D/2}{D/2 + c} + 1 \right) c \Delta w = (\alpha - 1) \pi D \frac{\Delta D}{2} \quad (10)$$

where $\alpha = \rho_{rust}/\rho_{steel}$ (the relation between the densities of the rust product and the steel) depends on the type of corrosion products. Typical values are 2 to 4. By inserting the above-mentioned data one obtains for this case $\gamma = 1.4$ to 4.2 in good agreement with the experimental results described by [1].

5. Conclusions

Modelling of corrosion crack initiation and corrosion crack evolution is presented with special emphasis to modelling of the crack evolution. Experiments and FEM analysis seems to support that the function between the reduction of the rebar diameter ΔD_{bar} and the corresponding increase in crack width Δw_{crack} in a given time interval Δt measured on the surface of the concrete specimen can be approximated by a linear function.

Acknowledgments

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